MATH-820 Calculus of Variations and Optimal Control

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The major purpose of this course is to present theoretical ideas and analytic and numerical methods to enable the students to understand and efficiently solve optimization problems.

Core Contents: The Finite dimensional problem: The free problem. Equality constrained problem. The inequality constrained problem, Newton's Method. The basic theory of the calculus of variations: Introduction, Some examples. Critical point conditions. Additional necessary conditions. Miscellaneous results. Sufficiency theory. Several dependent variables.Optimal control, The minimal time problem, Unconstrained Reformulations. Constrained calculus of variations problems. Kuhn- Tucker reformulation. Numerical methods and results.Kuhn-Tucker method. Introduction to fractional calculus. Fractional calculus of variations, Fractional Euler–Lagrange equations

Course Outcomes: Students are expected to understand:

- The theory of the calculus of variations.
- The optimal control problems.
- Numerical methods and results for optimization.
- Fractional calculus of variations.

Text Book:

1. John Gregory, Cantian Lin, Constrained Optimization in the Calculus of Variations and Optimal Control Theory, Springer (1992).
2. Ricardo Almeida, Dina Tavares Delfim F. M. Torres, (RAD) The Variable-Order Fractional Calculus of Variations, Springer 2019.

Reference Books:

1. M. D. Intriligator, Mathematical Optimization and Economic Theory, Siam

(2002).
2. Pablo Pedregal, Optimization and Approximation, Springer (2017)
3. Daniel Liberzon, Calculus of Variations and Optimal Control Theory, PRINCETONUNIVERSITY PRESS, (2012).

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1,1.2	The Finite dimensional problem: The free problem, The equality constrainedproblem.
2	1.3, 1.4	The inequality constrained problem, Newton's Method.
3	2.1-2.3	The basic theory of the calculus of variations: Introduction, Some examples
4	2.3	Critical point conditions.
5	2.4, 3.1	Additional necessary conditions, Miscellaneous results
6	3.2	Sufficiency theory.
7	3.3	Several dependent variables.
8	4.1	Optimal control: A basic problem
9	**Mid Semester Exam**	
10	4.2, 5.1	The minimal time problem: An example of abnormality. UnconstrainedReformulations: The optimal control problems.
11	5.2,5.3	Constrained calculus of variations problems, Kuhn-Tucker reformulation

12	6.1	Numerical methods and results: The basic Problem in calculus of variations
13	6.2	Numerical transversality conditions for general problems
14	6.3	Kuhn-Tucker method
15	2.1,2.2 (RAD)	Introduction to fractional calculus
16	3.2	Fractional calculus of variations, Fractional Euler–Lagrange equations
17		Review
18	**End Semester Exam**	